Title: Exponential Growth And Decay

Brief Overview:

Using activity sheets and the TI-82/83 students will be able to take statistical information and list independent and dependent variables, plot data points, and find and graph the exponential model that best fits. Real-life examples will be exponential growth and decay.

Links to NCTM Standards:

- **Mathematics as Problem Solving**
  Students will apply knowledge gathered from graphical information to apply to real-world situations.

- **Mathematics as Communication**
  Students will be able to translate information either given or collected to apply exponential functions through written assessments.

- **Mathematics as Reasoning**
  Students will be asked to predict exponential growth or decay based on the data that had been given or collected.

- **Mathematical Connections**
  Students will be able to connect real-life data to the algebraic representation of exponential functions.

- **Algebra/Functions**
  Students will represent real-life situations by using equations and graphs to interpret data.

- **Statistics**
  Students will use data charts to summarize data.

- **Pre-Calculus**
  Students will discover exponential equations from real-life data.

Grade/Level:

Grade 9-12; Algebra II and Pre-Calculus

Duration/Length:

Two to three class periods (variable)

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Graphing linear and quadratic functions
- Use of rational exponents
- Exponential growth and decay problems without the use of the TI-82/83

Objectives:

Students will:

- collect, organize, and enter data.
- write an exponential model from the data.
- apply reasoning to other disciplines.

Materials/Resources/Printed Materials:

- TI-82/TI-83
- Students worksheets
- Overhead (optional)

Development/Procedures:

Pre-Lesson Activity

- Start the lesson by reviewing linear and quadratics equations on the graphing calculator.
- Review with students that in the function $f(x) = ab^x$, b is the growth factor when $b > 1$ and a decay factor when $0 < b < 1$.

Lesson Activity

- Teacher/Student Guide -- includes using the TI-83 to plot data points and to find the exponential function.
- Activity Sheet -- allows students to apply and analyze solutions to other exponential problems.
- Assessment -- includes varying types of math problems.
- Long-Term Activity -- is an application to the students' real-life situation.

Extension/Follow Up:

Have students find graphs of exponential functions. They should identify the graphs as growth or decay.

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TEACHER/STUDENT GUIDE

PLOTTING DATA POINTS (general directions for activity)
Use TI-83 graphing calculator to plot points of exponential functions.

Press [Y=] menu clear or turn off all functions

Press [FORMAT] select *Grid Off*
Press [MODE] [▼] 3 times to get to *function*
Press [ENTER] to set. Calculator will now be in function mode

1) Press [STAT], press [1] (*Edit.*).

2) Move the cursor so that the **L1** or **L2** is highlighted, press [CLEAR], press[ENTER].

3) Enter data in the chosen list.

4) To display the data press [2nd], press[STAT PLOT], press [1].

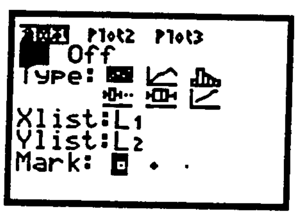
5) Press [ENTER] to select On, which turns on *plot 1.*
Press [▼] press [ENTER] to select (*scatter plot).*
Press [▼] press [ENTER] to specify list 1 for plot 1.
Press [▼] press [ENTER] to specify list 2 for plot 1.
Press [▼] press [ENTER] to select the mark for each data point on the scatter plot.

6) To display the data, press [ZOOM], press [*9*] (*Zoom Stat*)

PROBLEM:
The price of a new car is $20,000.
The depreciation value is 15%.

The depreciation formula is:
$y = 20,0000(.85)^x$
when x = number of years
y = value of the depreciation

1. Use the given chart to list the data in the **L1** (years) and **L2**.
   Press the [STAT PLOT]
2. Plot 1 [ENTER]

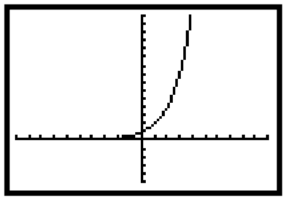


3. [ZOOM] #9 Zoom Stat
4. [GRAPH]
5. Predict the value of the car in 10 years.
6. Check your prediction on the calculator.

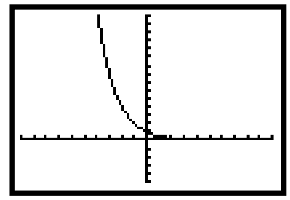
YEARS	DEPRECIATION
0	20,000
1	17,000
2	14,450
3	12,283
4	10,440
5	8,874
6	7,543
7	6,411
8	5,449

Using the TI-83 to Find a Mathematical Model for an Exponential Equation.

Exponential Model $y = ab^x$



when $a > 0$ and $b > 1$



when $a > 0$ and $0 < b < 1$

This is called an **exponential regression** model. You will need to plot data points using the STAT menu.

1) Plot Data Points as described on previous worksheet.

2) Press [**STAT**], press [ ] (*Calc*).

Press [**0**] to find an exponential model (*ExpReg* ($y = ab^x$)).

Press [**Enter**]. The a and b values of the exponential model are now displayed.

You are now ready to enter the equation into the Y=menu.
Two methods will be described.

3) Write the equation on a worksheet.
Press [**Y=**]. Enter the equation.

or Press [**Y=**], press [**VARS**], press [**5**] (*Statistics*), press [ ] 2 times (*EQ*), press [**1**] (*Reg EQ*).
The equation will appear on the Y= screen.

4) Press [**GRAPH**].
The graph of the exponential model will appear along with the data points.

A quick way of finding other independent and dependent values.

5) Press [**2nd**], press [**TABLE**].
You may move your cursor in either the column to locate the value you are seeking.

Sometimes scrolling takes too long. In this case you may set your table to begin at a different value.

6) Press [**2nd**], press [**TBLSET**] and key in the starting x-value at TbleStart = $\boxed{}$.
Now repeat step 5.

ACTIVITY SHEET:

The town planners designed the town for an optimal growth of 8% per year. The present school construction will serve a population of 200,000.

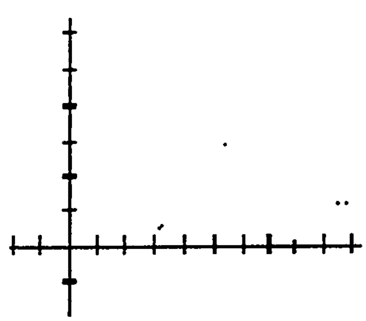
Below is a table representing the growth from 1980 to 1986.

1. List given data in your calculator.

2. Find and write the model of exponential regression.

3. Sketch the graph on the axis provided.

YEAR	POPULATION
0	50,000
1	54,000
2	58,320
3	62,986
4	68,024
5	73,466
6	79,344



4. Predict what the population will be in the year 2000.

5. In what year will the population be doubled?

6. In how many years will the town need to build more schools to accommodate the population growth of 200,000?

ACTIVITY SHEET - KEY

The town planners designed the town for an optimal growth of 8% per year. The present school construction will serve a population of 200,000.

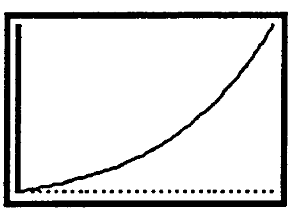
Below is a table representing the growth from 1980 to 1986.

1. List given data in your calculator.

2. Find and write the model of exponential regression.
   $Y = 50000.02129114 * 1.0799997358906^x$

3. Sketch the graph on the axis provided.

YEAR	POPULATION
0	50,000
1	54,000
2	58,320
3	62,986
4	68,024
5	73,466
6	79,344



4. Predict what the population will be in the year 2000.
   The population will be 233,047.

5. In what year will the population be doubled?
   The 9th year will be 1989

6. In how many years will the city need to build more schools to accommodate the population growth of 200,000?
   In 18 years

LONG TERM ACTIVITY (OPTIONAL)

Let us assume that the annual cost of a college education is increased by an exponential function.

a. Collect data from at least 3 colleges that gives the tuitions for a 5 year period.

b. Use a table to list your data.

c. Find the line of best fit.

d. Find the growth factor.

e. Give the function for the tuition.

f. Estimate what the tuition will be when you are ready to attend.

g. Which one will represent the best buy? Explain your answer.

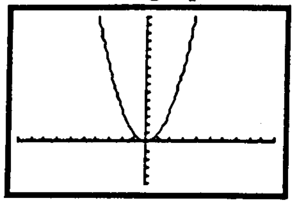
LONG TERM PROJECT-(OPTIONAL)

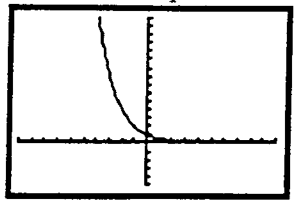
SCORING: Extra credit ---each item (a to g) worth 1 pt.

Name:______________________________ **ASSESSMENT**

Circle the correct answers for #'s 1 and 2.

1. Select the graph that represents an exponential growth.


a.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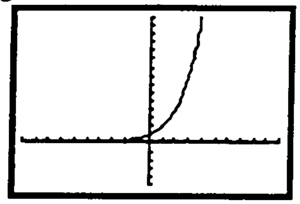
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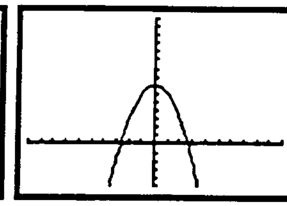

d.

2. Select the function that has a decay factor of .75 ?

a. $y = .75\,(.3)^x$ b. $y = .75(300)^x$ c. $y = 0.75x$ d. $y = 300(.75)^x$ e. $y = 300x^{0.75}$

Use the data below to answer question #3.

X (Independent Variable)	0	1	2	3	4	5
Y (Dependent Variable)	40	42	44.1	48.1	50	53

3a. Can the data be described as exponential function?

b. Use a graph, an equation or a logical argument to justify your previous answer.

4. Give two real-life situations where exponential growth can or has occurred.

5. Give two real-life situations where exponential decay can or has occurred.

The function $y = 60{,}000(1.035)^x$ models the growth of a town. The population seems to be increasing at a rate of 3.5% per year. The number of years is represented by x and y represents the population. The town was chartered in 1980, therefore in 1980, $x = 0$ and the population of the town was 60,000.

6a. In what year will the population exceed 150,000?

b. If the town continues to grow at this rate the population in the year 2000 will be

______________________________.

Name:____________________________ **ASSESSMENT**

Oceanography The intensity of sunlight decreases as you descend in the ocean. The table below shows the percent of sunlight found below sea level beginning at a depth of 20 feet. (The model you will be asked to find is accurate for depths of 20 to 600 feet.)

Feet (Below Sea Level)	Percent of Sunlight
20	12
40	9.36
60	7.2
80	2.6
100	1.5
110	1.2
150	.8
200	.12
250	.035
300	.01

Use the graphing calculator to draw a scatter plot of the data.
Use **ExpReg** to determine the exponential equation that best fits the data.

7. Write the exponential equation of the curve of best fit.

8. Does the equation represent exponential growth or decay?

9. Find the percent of sunlight found 70 ft. below sea level.

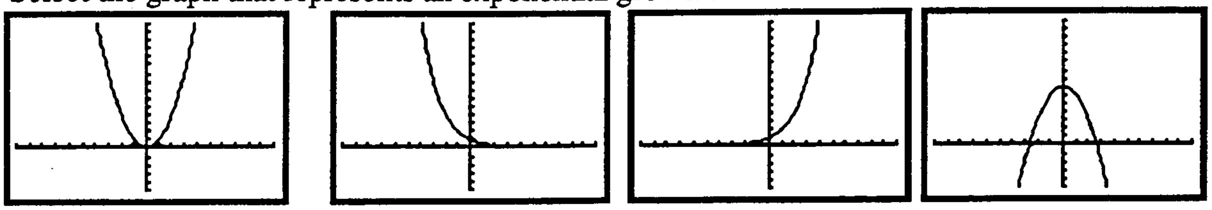
10. What percent of sunlight could a deep-sea diver expect to find at 299 ft. below sea level?

11. Looking at your table, explain why the exponential model is not accurate at sea level.

Name:________________________ **ASSESSMENT-KEY**

Circle the correct answers for #'s 1 and 2.

1. Select the graph that represents an exponential growth.



a. b. c. d.

2. Select the function that has a decay factor of .75 ?

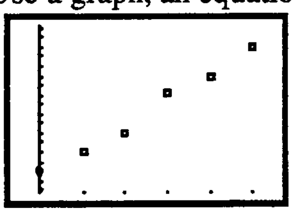
a. $y = .75(.3)^x$ b. $y = .75(300)^x$ c. $y = 0.75x$ d. $y = 300(.75)^x$ e. $y = 300x^{0.75}$

Use the data below to answer question #3.

X (Independent Variable)	0	1	2	3	4	5
Y (Dependent Variable)	40	42	44.1	48.1	50	53

3a. Can the data be described as exponential function? ***YES***

b. Use a graph, an equation or a logical argument to justify your previous answer.



OR

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ExpReg
 y=a*b^x
 a=39.80663083
 b=1.059319791
```

OR ***LOGICAL ARGUMENT***

4. Give two real-life situations where exponential growth can or has occurred.

Scoring Key:

3	*This response includes two accurate answers.*
2	*This response includes one accurate answer.*
1	*This response is incomplete.*
0	*No response*

5. Give two real-life situations where exponential decay can or has occurred.

Scoring Key:

3	*This response includes two accurate answers.*
2	*This response includes one accurate answer.*
1	*This response is incomplete.*
0	*No response*

The function $y = 60{,}000(1.035)^x$ models the growth of a town. The population seems to be increasing at a rate of 3.5% per year. The number of years is represented by x and y represents the population. The town was chartered in 1980, therefore in 1980, $x = 0$ and the population of the town was 60,000.

6a. In what year will the population exceed 150,000?
2007

b. If the town continues to grow at this rate the population in the year 2000 will be ***119,387***
________________________.

Name:____________________________ **ASSESSMENT-KEY**

Oceanography The intensity of sunlight decreases as you descend in the ocean. The table below shows the percent of sunlight found below sea level beginning at a depth of 20 feet. (The model you will be asked to find is accurate for depths of 20 to 600 feet.)

Feet (Below Sea Level)	Percent of Sunlight
20	12
40	9.36
60	7.2
80	2.6
100	1.5
110	1.2
150	.8
200	.12
250	.035
300	.01

Use **ExpReg** to determine the exponential equation that best fits the data.

7. Write the exponential equation of the curve of best fit.

$y = 24.4292752(.9743647854)^x$

8. Does the equation represent exponential growth or decay?

exponential decay

9. Find the percent of sunlight found 70 ft. below sea level.

3.9666%

10. What percent of sunlight could a deep-sea diver expect to find at 299 ft. below sea level?

.01037%

11. Looking at your table, explain why the exponential model is not accurate at sea level.

At sea level, one would find 100% sunlight . When x = 0 , that represents sea level, the table shows that only 24.429% sunlight is found, therefore this model is not accurate at sea level.

Scoring Key:

3	*Response is complete and gives a clear verification of answer*
2	*Response is correct but verification unclear*
1	*Response is incorrect but an attempt to verify was made*
0	*No response*